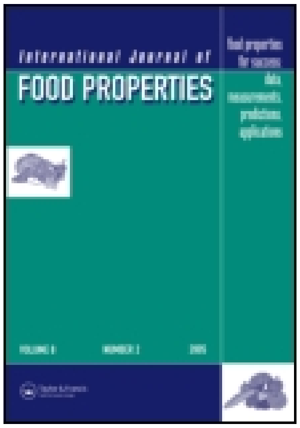

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THERMAL PROPERTIES OF CHEDDAR CHEESE: EXPERIMENTAL AND MODELING

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THERMAL PROPERTIES OF CHEDDAR CHEESE: EXPERIMENTAL AND MODELING

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ABSTRACT

Thermal properties (thermal conductivity, thermal diffusivity and heat capacity) of Cheddar cheese were measured as a function of cheese age and composition. The composition ranged from 30–60% moisture, 8–37% fat, and 22–36% protein (wet basis). The thermal conductivity and heat capacity ranged from 0.354–0.481 W/m °C and from 2.444–3.096 kJ/kg °C. Both thermal conductivity and heat capacity increased with moisture and protein content and decreased with fat content. The thermal diffusivity ranged from 1.07×10^{-7}–$1.53 \times 10^{-7}\,m^2/s$. There was no significant relationship between thermal diffusivity and moisture, fat and protein content of cheese. No statistically significant effect (at the 10% level) of age (0 to 28 wk) on thermal properties was observed. Models predicting thermal properties as a function of cheese composition were developed and their predictive ability was compared with that of empirical models available in the literature. In addition, several theoretical thermal conductivity models were evaluated for their usefulness with Cheddar cheese. Published thermal conductivity models cannot accurately predict (mean error was from 3.4 to 42%) the thermal conductivity of Cheddar cheese.

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INTRODUCTION

There is now a large literature that presents empirical data of thermal properties of food, as well as literature on composition-based approach to predict thermal properties of food. Examples of literature for thermal conductivity values and prediction of food are Sweat et al. [1–5], Heldman and Singh [6], Baghe-Khandan et al. [7], Choi and Okos [9], Wallapapan et al. [10], Zuritz et al. [11], Srilomsak [12], Rahman [13], etc. Mathematical models by Long [14], Lentz [15], Kopelman [16], Poppendick et al. [17], and Rahman [13] have discussed this property.

Similar to thermal conductivity, heat capacity prediction can be approached empirically and theoretically. Empirical models of heat capacity prediction are not as varied as models predicting other thermal properties. Heat capacity has been commonly modeled with an equation of the form of $C_p = C_1 + C_2 \times W$, where C_p is heat capacity (J/kg °C), C_1 and C_2 are product-specific parameters, and W is percent moisture (wet basis) [18–20]. The theoretical model frequently used to predict heat capacity of a mixture of components is based on weighted averages of the heat capacities of the individual components [6, 9, 4, 10, 21, 11, 22, 13].

Literature on thermal diffusivity values of foods is less comprehensive because it is more difficult to determine thermal diffusivity values than other thermophysical properties of foods. In most studies on thermophysical properties of foods, the value of thermal diffusivity was calculated using the following relationship:

$$\alpha = k/(\rho \times C_p) \tag{1}$$

where α is thermal diffusivity (m^2/s), k is thermal conductivity (W/m °C), ρ is density (kg/m^3), and C_p is heat capacity (J/kg °C) [23–25, 9, 4, 21, 13].

Amid those vast literatures on thermal property values and predictions of food, there is very little known for thermal properties of cheese [26, 27]. It is also known that because of the complex nature of food, it is hard to be sure that one can get a good prediction of these basic properties from other general models using other type of food. The preliminary research comparing the thermal properties of several types of food products using the Choi and Okos models and the published experimental values showed that their models could predict well the thermal properties of liquid food, but their accuracy for solid foods were poor [9].

Cheddar cheese is particularly interesting because it is intermediate in textural properties between crumbly cheeses and plastic cheeses [28]; and in addition, it is a relatively homogenous product compared to other cheeses. Accurate data on thermal properties of Cheddar cheese as a function of composition are needed to better predict functional properties such as meltability. This would help develop guidelines for appropriate formulations so that cheese can be incorporated into other food products.

The specific objectives of this study were to:

1. experimentally determine thermal properties (thermal conductivity, thermal diffusivity, and heat capacity) of Cheddar cheese as a function of composition (fat, protein and moisture content) and age.
2. develop empirical models to predict thermal properties of Cheddar cheeses based on their composition.
3. compare the experimental values with those predicted using the published empirical models and theoretical models.

MATERIALS AND METHODS

Cheese

Cheddar cheeses of different composition were manufactured at the Center for Dairy Research (CDR), University of Wisconsin-Madison. They ranged from 33.7 to 57.7% for moisture content (wet basis), 7.8 to 36.7% for fat content, 22.4 to 35.0% for protein content, and 0 day to 28 weeks for age. Moisture content was determined in a vacuum oven at 100°C for 24 h [29], fat by the Babcock procedure [30], protein by the Kjeldahl method [31]. Density was measured using the simple displacement method [32]. The assays were done in three replications. There were no significant pores or voids in the cheeses used in this study.

Thermal Conductivity Measurement

Thermal conductivity of cheese was measured using the line heat source technique described by Murakami et al. [33]. The details of the basic theory and mathematical derivation behind the use of the line heat source probe have been discussed previously by Hopper and Lepper [34] and Nix et al. [35]. Murakami et al. [33] gave a comprehensive discussion about the design of thermal conductivity probe and its limitations. A line heat source probe was designed and fabricated for this study. The probe was consisted of a 3.8 cm-long stainless steel needle tubing (dia: 0.635 mm), enclosing an insulated constantan heater wire (dia: 0.076 mm), and an insulated E-type thermocouple wire (dia: 0.051 mm), which measured temperature at the mid-point along the needle.

The thermal conductivity measurement was conducted by inserting the probe into the middle of a cheese block, then heat was supplied by a heater for two minutes, and its time-temperature data were monitored every two seconds. After the few seconds of heating, the temperature started to rise linearly from about 21 to 25°C with natural logarithm of time and the slope of this portion was determined. The sample's thermal conductivity (k) was determined by dividing the slope (ΔTemperature/Δln(time)) with a probe factor (G). The probe factor was determined by running the experiment with a calibration sample (water gelled with 0.8% agar) with known thermal conductivity value of 0.626 W/m °C. The experimental error of this probe factor was about 6%. The probe was further tested by measuring other materials of known k-values, such as NaCl brine (30% w/v and 15% w/v; gelled with 0.5% agar) and Glycerol (100%) and their k-values were within 6% of their published values.

The correlation coefficient (R^2) of the linear portion of the temperature-ln (time) curve was also determined. A curve with an R^2 of less than 0.98 was considered to contain excessive measurement errors and was discarded. On average, less than one out of five replications was discarded for this reason. The overall experimental errors of thermal conductivity measurement were usually less than 5%.

Heat Capacity Measurement

During the course of this study, three DSC (differential scanning calorimeter) were used to measure the heat capacity of Cheddar cheese. They were manufactured by Netzsch (Model DSC-200), by Perkin Elmer (Model DSC-7) and by TA Instruments (Model 2920 Modulated DSC). This, of course, is not an ideal situation but had to be done due to technical difficulties during the course of the study. However, extreme precaution was taken to ensure the consistency of the results.

Before each test, the equipment was calibrated according to the manufacturer's specification. After calibration, the base line was established by running the program with no sample present. Weighed empty aluminum sample pans were placed in both sample and reference holders and scanned at a programmed heating rate of 5°C/min over a selected temperature interval (0–100°C). In order to give the initial and final transient behavior time to disappear, the temperature program was started at -10°C and ended at 110°C [36]. The procedure was repeated with a known mass of sapphire standard and samples of cheese [25–30 mg]. DSC thermoforms typically resulted in seemingly random, very noisy observations between 0 and 50°C. Christenson et al. [37] observed a similar erratic behavior and attributed it to endothermic peak caused by melting fat. For this reason the average heat capacity values from 60 to 90°C was considered as the value of the measurement. Three samples were used for each experiment and the range of experimental errors was 5–15%. Due to the presence of moisture, hermetically sealed sample pans were used. A pan sealer was used to seal the sample pan to prevent moisture losses occurring during heating. After completion of each run the sample pan was reweighed to ensure there was no loss of sample mass during the run.

Thermal Diffusivity Measurement

This study measured thermal diffusivity of Cheddar cheese, as well as comparing two different methods for obtaining this property. The first one was using an apparatus described in Dickerson [38]. It consisted of an agitated water bath at 60°C in which a steel cylinder (20 cm in length and 4 cm in diameter), insulated with rubber corks on both ends, containing the cheese sample was immersed. Type T thermocouple (dia: 0.051 mm) was soldered to the outside surface of the tube monitoring the surface temperature of the sample. The center temperature of the sample was measured by inserting a stainless steel needle tubing (OD: 0.889 mm; L: 7.4 cm), containing a thin type T thermocouple, through the center of the upper rubber cork. After a period of equilibration between the sample temperature and the water temperature, the heater was turned on for at least one hour or until the sample center temperature increases at the constant, water-bath heating rate (about 0.67°C/min). The measurements were repeated three times for each experiment.

The second method for determining thermal diffusivity values was using time-temperature history method. The underlying principle and the mathematical theory behind the unsteady state heat transfer for infinite cylinders can be found in Charm [39]. He has solved the unsteady state heat transfer for an infinite cylinder at a uniform initial temperature and exposed to a constant-temperature environment as follows:

$$\frac{T_a - T}{T_a - T_i} = \frac{2}{R}\sum_{n=1}^{\infty} \frac{J_0\left(B_n \frac{r}{R}\right)\left[\exp\left(\frac{-B_n^2 k\theta}{\rho C_p R^2}\right)\right]}{\left(1 + \frac{k^2 B_n^2}{h^2 R^2}\right)\frac{B_n}{R}\left[J_1(B_n)\right]} \tag{2}$$

where, T_a = ambient temperature (°C), T_i = initial temperature (°C), T = temperature at time θ (°C), R = radius of the cylinder (m), J_0 (X) = zero order of first kind Bessel function of X, B_n = root of m $J_0(B_n) = B_n J_1(B_n)$, where $m = hR/k$, r = distance from the center (m), k = thermal conductivity (W/m °C), θ = time (s), r = density (kg/m^3), C_p = heat capacity (J/kg °C), h = surface heat transfer coefficient (W/m^2 °C), $J_1(X)$ = first order of first kind Bessel function of X.

The experiments were conducted by cutting cheese blocks with a cork borer to a cylindrical shape of 46–48 mm in diameter and 70–80 mm in height. The cheese samples were equilibrated to room temperature inside a plastic bag to avoid any moisture and fat loss. In this experiment, the heat transfer was assumed to occur only in one dimension, i.e. in radial direction. Thus special care was taken to insulate the ends of the cheese sample using Styrofoam. The insulation was held in place with metal rods to keep them from sliding down when the cheese melted (Figure 1). Three thermocouple probes were used to measure the temperatures across the cheese samples:

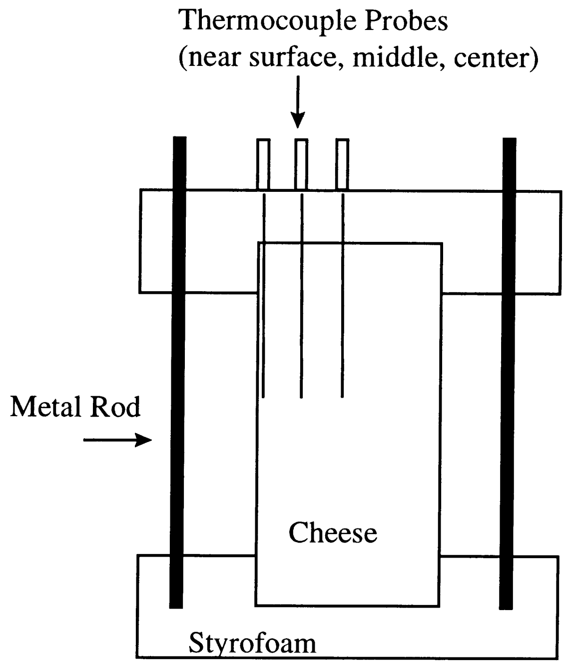

Figure 1. Schematic of a temperature profile apparatus.

center, middle, and near surface temperatures. Then, the apparatus was put inside a pre-heated forced-convection oven set at 55–57°C for 30 min. The velocity of hot air inside the oven was measured by an airflow meter and was assumed to be constant throughout the study. Parameters measured were the diameter of the sample, the exact location of the probes, initial temperature of the sample, oven temperature, and the time-temperature data during heating. A computer program was written in GAUSS (Aptech Systems, Inc., Maple Valley, WA) to calculate the thermal diffusivity of Cheddar cheese using Eq. (2). The value of surface heat transfer coefficient (h, $W/m^2\,°C$) was determined from Nusselt number calculation as shown in Marschoun [40]. Another computer program was written in GAUSS to predict the temperature profiles at different locations of the samples. A listing of the computer program can be found in Marschoun [40].

Thermal Properties Modeling

Several empirical models were built using simple multivariate regression method (Minitab 10.5). The empirical models were examined against theoretical models and other published empirical models. Nine theoretical models for thermal conductivity prediction (Table I) were chosen for this test. In the Maxwell-Eucken model (Eq. 3), the foods tested were assumed to be a system

Table I. Thermal Conductivity Theoretical Models

Model	Model No.	Equation	Variable Notation*	Eq.
Maxwell-Eucken	ME	$k = k_c\left[\frac{1-\left(1-A\frac{k_d}{k_c}\right)B}{1+(A-1)B}\right]$	$A = 3k_c/(2k_c + k_d)$ $B = V_d/(V_c + V_d)$ or volume fraction of the dispersed phase (X_d^v) V_c = volume of continuous phase V_d = volume of dispersed phase	(3)
Kopelman #1	K1	$k = k_c\left[\frac{1-M^2}{1-M^2(1-M)}\right]$	M^3 is the volume fraction of solids or discontinuous phase in the product	(4)
Kopelman #2	K2	$k = k_c\left[\frac{1-Q}{1-Q(1-M)}\right]$	Q is $M^2(1 - k_d/k_c)$	(5)
Kopelman #3 (parallel)	K3	$k_{\|\|} = k_c\left[1-P\left(1-\frac{k_d}{k_c}\right)\right]$	P is the volume fraction of the discontinuous phase	(6)
Kopelman #4 (perpendicular)	K4	$k_{\perp} = k_c\left[\frac{k_d}{Pk_c + k_d(1-P)}\right]$		(7)
Poppendick #1	P1	$k = \rho\sum_{i=1}^{3}\frac{k_i W_i}{\rho_i}$	ρ = density of the mixture W_i = mass fraction of the ith constituent	(8)
Poppendick #2	P2	$k = \frac{1}{\rho\sum_{i=1}^{3}\frac{W_i}{k_i\rho_i}}$		(9)
Series	SE	$\frac{1}{k} = \sum_{i=1}^{n}\frac{X_i^v}{k_i}$	$X_i^v = \frac{X_i^w/\rho_i}{\sum(X_i^w/\rho_i)}$ X_i^v = volume fraction of the ith constituent X_i^w = mass fraction of the ith constituent	(10)
Parallel	PL	$k = \sum_{i=1}^{n} k_i X_i^v$		(11)

Note:
* Common variable notations are presented below and the same notation represents the same meaning.
k = thermal conductivity of mixture.
k_c = thermal conductivity of continuous phase.
k_d = thermal conductivity of dispersed phase.
k_i = thermal conductivity of the ith constituent.
ρ_i = density of the ith constituent.

with a continuous and dispersed, which was assumed to be spherical in shape, phase. In the application of the model to Cheddar cheese, fat was assumed to be the dispersed phase and Cheddar cheese with 0% fat content was assumed to be the continuous phase. This model predicts the thermal conductivity of a mixture as a function of fat content. Several fixed values need to be determined before using the model: (a) k_d, thermal conductivity of fat, was 0.114 W/m °C and fat density was set to 0.915 g/cm (8), (b) thermal conductivity of Cheddar cheese with 0% fat (starch was used as fat replacer) was measured to be 0.52 W/m °C and the measured value of its density was 1.12 g/cm^3. The Kopelman model (Eq. (4)) uses the same parameters and variables as Maxwell-Eucken, except that it does not include a value for the thermal conductivity of the dispersed phase.

The models built in this study were compared to other empirically built models from literature to predict thermal property values of out-of-sample data. Out-of-sample data is separate data collected from this study but was not used when constructing the models. Models from Choi and Okos [8] were used for all three thermal properties, Sweat [5] for thermal conductivity, Hermans [41] for thermal diffusivity, and Heldman and Singh [6] for heat capacity.

RESULTS AND DISCUSSION

Effect of Composition and Age on Thermal Properties of Cheddar Cheese

There was a negative correlation between fat content and moisture content ($R^2 = 0.86$) as also observed by Sweat and Parmelee [26]. This means that changes in one of the cheese components bring about related changes in other components. This important fact was utilized when building the models later on this study.

The thermal conductivity of Cheddar cheese tested in this study ranged from 0.354–0.481 W/m °C with high quality data as measured by their coefficient of variation (CV) of less than 5%. Table II shows the thermal properties values of Cheddar cheese with its chemical composition. Thermal conductivity of Cheddar cheese increased with moisture content, decreased with fat content and increased with protein content. This conclusion was drawn from plotting the thermal conductivity values against its respective cheese composition and from statistical analyses using univariate regression method (Table III). All of these trends agreed with theory and previous research for other products.

The heat capacity of Cheddar cheese tested in this study ranged from 2.444–3.096 kJ/kg °C with CV ranged from 5–15%. The heat capacity of Cheddar cheese increased with moisture and protein content but decreased with fat content. Table III shows a highly significant correlation between heat capacity of Cheddar cheese and moisture, fat, and protein content.

Table II. Composition of Cheddar Cheese with Their Thermal Properties Values

No.	Moisture Content (%)	Fat Content (%)	Protein Content (%)	Thermal Conductivity (k, W/m °C)	Thermal Diffusivity ($\alpha \times 10^{-7}$, m^2/s)	Heat Capacity (C_p, kJ/kg °C)
1	33.65	34.94	26.09	0.354	1.18	N/A
2	34.80	35.15	26.09	0.356	1.09	N/A
3	44.23	23.40	28.36	0.391	1.17	2.902
4	47.43	17.20	29.75	0.423	1.17	2.761
5	50.00	17.00	30.28	0.432	1.10	2.923
6	54.92	9.40	32.55	0.472	1.34	2.970
7	37.94	32.25	26.72	0.388	1.12	2.451
8	45.58	19.35	28.02	0.428	1.16	2.601
9	50.15	10.20	31.04	0.465	1.17	2.804
10	45.44	21.65	30.28	0.425	1.21	2.640
11	41.15	24.00	28.47	0.422	1.23	2.796
12	39.42	25.80	28.06	0.410	1.18	2.609
13	47.09	17.16	30.12	0.448	1.25	2.940
14	37.00	N/A	N/A	0.369	1.14	2.639
15	36.69	32.25	N/A	0.353	1.24	2.340
16	33.82	33.25	N/A	0.338	1.16	2.429
17	36.20	34.38	N/A	0.366	1.26	2.414
18	40.54	33.25	N/A	0.381	1.27	2.524
19	36.02	34.00	25.06	0.359	1.21	2.444
20	38.00	29.80	N/A	0.356	1.20	2.481
21	47.20	26.60	N/A	0.403	1.23	2.660

Note: N/A = data not available.

The experimental data of thermal diffusivity of Cheddar cheese using the Dickerson method were hampered by high measurement errors, which were as high as 25%. The thermal diffusivity of Cheddar cheese tested in this study ranged from 1.07×10^{-7}–1.53×10^{-7} m^2/s. Univariate regression performed between thermal diffusivity and various cheese components did not reveal any significant relationship between thermal diffusivity and moisture, fat and protein content of cheese (Table III).

Large measurement errors resulting from the use of the Dickerson method to determine thermal diffusivity suggest this method is not appropriate to measure thermal diffusivity of Cheddar cheese. To verify this limitation, an experiment was conducted to measure thermal diffusivity of distilled water gelled with 0.3% agar. The value was found to be 1.55×10^{-7} m^2/s which was in good agreement with results reported by Dickerson and Read [42] and Rizvi et al. [43]. Large experimental errors may be due to the following reasons: 1) Discontinuity form of the sample inside the tube because it was almost impossible to have a single 20 cm-long piece of sample. Two cheese cylinders, about 10 cm-long each, were used as

Table III. Summary of Univariate Regressions of Thermal Properties of Cheddar Cheese on Its Components

Component	Coefficient	Standard Error	*t*-Stat
Thermal conductivity			
Moisture, %wb	0.00593	0.00057	10.374
Fat, %	−0.00453	0.00037	−11.969
Total protein, %	0.00593	0.00057	10.374
Heat capacity			
Moisture, %wb	0.02808	0.00443	6.327
Fat, %	−0.02154	0.00324	−6.638
Total protein, %	0.07064	0.01883	3.750
Thermal diffusivity			
Moisture, %wb	0.00265	0.00215	1.235
Fat, %	−0.00115	0.00168	−0.684
Total protein, %	0.01422	0.00754	1.884

Note: *t*-Stat higher than 2.05 shows that the result is statistically significant at 5% level.

the sample. 2) Non-uniformity of the sample shape. Depending upon the texture of the sample there was distortion from the cylinder shape when the cheese block was cut with cork borer. This effectively reduced the heat transfer area.

The thermal diffusivity values obtained by the time-temperature method are presented in Table IV. An ANOVA test could not be performed because of lack of replication from the Dickerson method. However, observing that in eight out of the nine cases for which data are available the Dickerson method yielded a value which is more than one coefficient of variation away from the value calculated using the time-temperature method strongly suggests the two methods yield different results. Because the coefficient of variation of the time-temperature method is significantly smaller than that of the Dickerson method, the time-temperature is considered to be the superior measurement technique.

Temperature history simulation was conducted to further assess the validity of time-temperature method. The simulated temperature histories agreed well with the experimental data (Figure 2). The percent errors of the experimental data and simulated data at three locations (center, middle, near surface) ranged from 0% to 15% with an average ranging from 0.6% to 11.6%. These deviations are presumably partially due to the fact that the thermal diffusivity was assumed to be constant in the calculation of the experimentally obtained values, while thermal diffusivity actually changes with temperature. Variations in environment conditions, such as oven temperature and hot air velocity inside the oven, during experiments could also

Table IV. Thermal Diffusivity Values of Ten Different Cheddar Cheeses Using Dickerson Method and Time-Temperature Method ($\alpha \times 10^{-7}\,m^2/s$)

		Time-Temperature	
No.	Dickerson	Mean ($n = 3$)	CV%
1	1.34	1.170	10.6
2	1.37	1.170	3.40
3	1.35	1.235	8.60
4	1.16	1.290	11.1
5	1.26	1.255	2.80
6	1.08	1.280	10.2
7	1.49	1.270	0.00
8	1.45	1.210	Not available
9	1.46	1.290	3.30
10	1.31	1.225	5.20

be responsible for the discrepancy. Dislocation of thermocouples due to melting cheese was an additional possible source of experimental error.

There was no significant difference ($p > 0.05$) in the moisture, fat, total protein content, and density values of all Cheddar cheeses over time. The values of thermal properties of Cheddar cheese over time are presented in

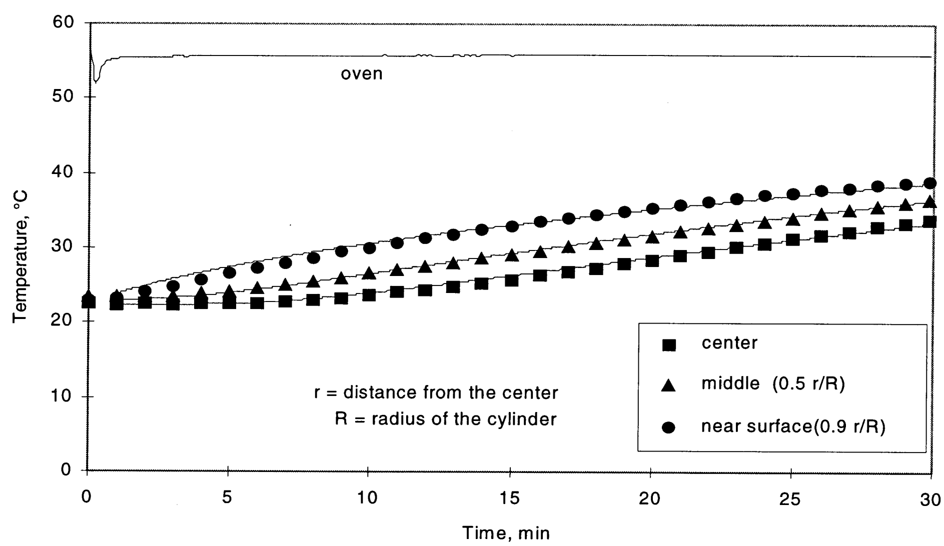

Figure 2. Experimental and simulated temperature history of Cheddar cheese during heating.

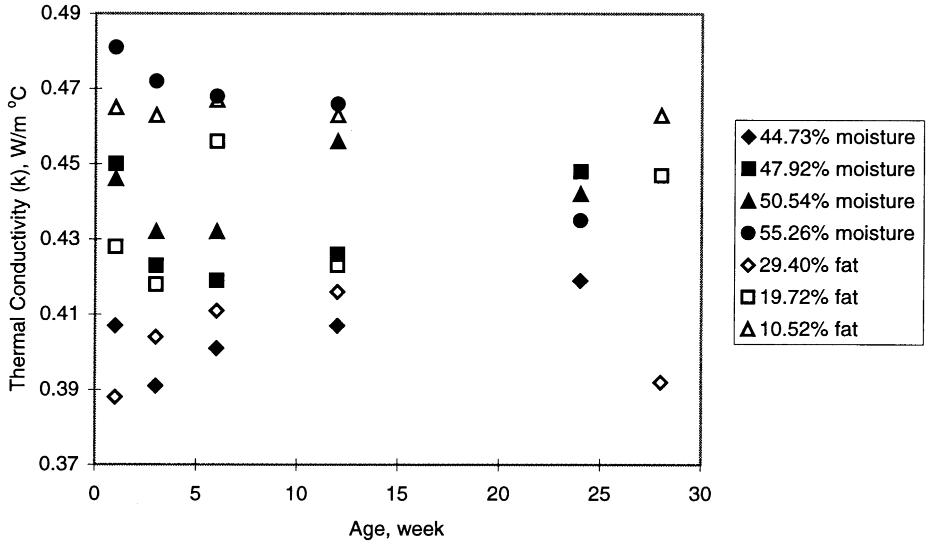

Figure 3. Thermal conductivity of Cheddar cheese as a function of age.

Figures 3–5. There was no significant difference ($p > 0.05$) in thermal conductivity of Cheddar cheese during aging. Differences in thermal diffusivity ($p > 0.1$) or in heat capacity ($p > 0.05$) over time were also not significant.

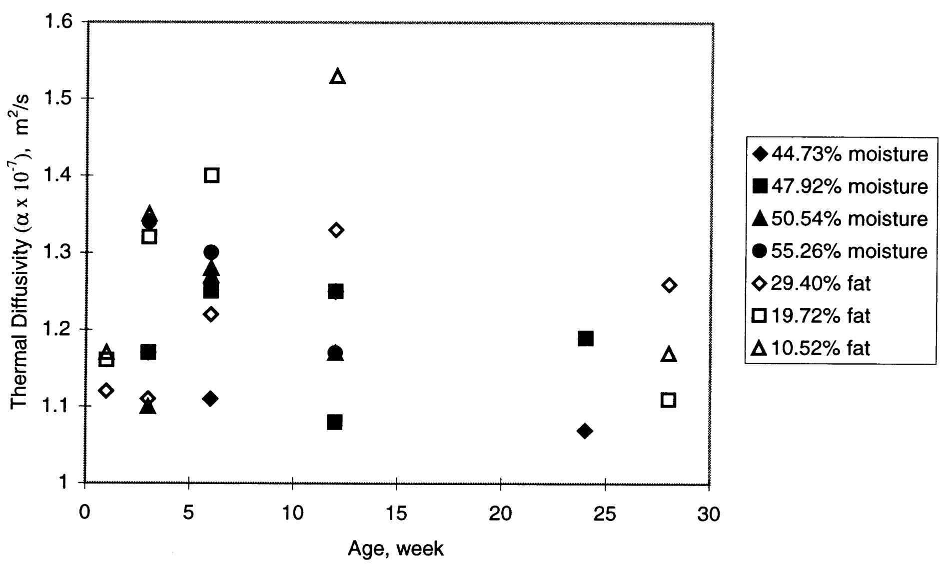

Figure 4. Thermal diffusivity of Cheddar cheese as a function of age.

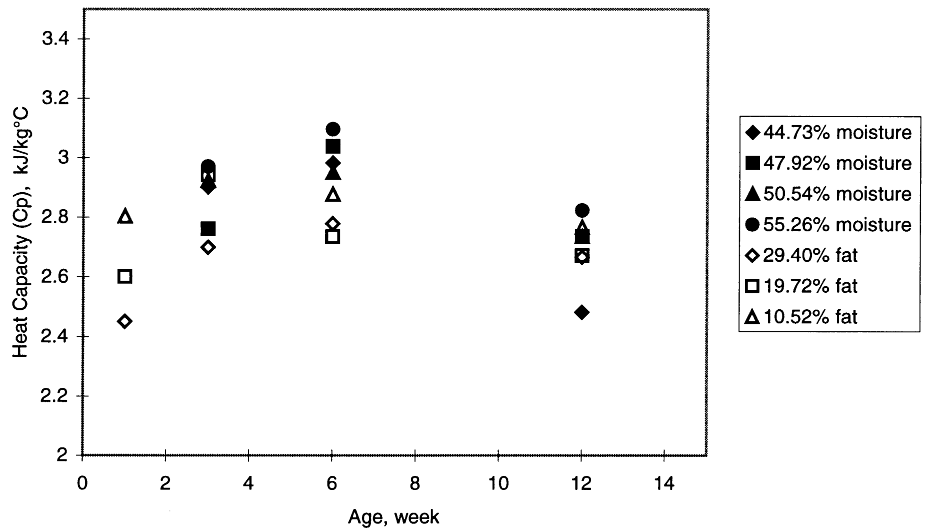

Figure 5. Heat capacity of Cheddar cheese as a function of age.

Modeling Thermal Properties of Cheddar Cheese

In general, cheese composition was found to have a significant influence on thermal properties. However, no systematic effect of cheese age on thermal properties could be found. Therefore, in the construction of empirical models age was not used as an explanatory variable. Data from cheeses at different aging times were considered as being in the same group and were thus clustered together. A univariate model cannot sufficiently explain the true relationship between the cheese components and its thermal properties. Therefore, multivariate regression method was used to build multivariate model that can predict thermal properties better and can address the question of causality adequately.

Three models were considered for thermal conductivity prediction of Cheddar cheese (Model k1, k2, and k3 in Table V). Model k2 and Model k3 had very low *t*-statistics for its interaction terms (example: 0.31, 1.02, and 0.61 for Model k2) and had to sustain a total of seven coefficient estimates with only 33 observations, which violates the rule-of-thumb of at least seven observations per explanatory variable. Model k3 also carried the same problem as Model k2. Therefore, being the most parsimonious specification, Model k1 was chosen over Models k2 and k3.

At first, Model k1 does not seem to agree with the univariate results from the previous section where thermal conductivity of Cheddar cheese has positive correlation with moisture content. In convention interpretation of a regression model, slope coefficient shows the change of the dependent variable when the respective independent variable is increased by one unit while

Table V. Multivariate Regression Models of Thermal Properties of Cheddar Cheese Based on Its Composition

No.	Model	n	R^2	SE
	Thermal conductivity, k (W/m °C)	33		
k1	$k = 0.445 - 0.00415F - 0.00116W + 0.00395P$		0.953	0.008
	(0.055) (0.00055) (0.00086) (0.0021)			
k2	$k = 0.87 - 0.011F - 0.0090W - 0.0099P + 0.000038F*W$		0.944	0.009
	(0.41) (0.0066) (0.0081) (0.015) (0.00016)			
	$+ 0.00021F*P + 0.00025W*P$			
	(0.00035) (0.00024)			
k3	$k = 0.40 - 0.00094F - 0.0074W + 0.013P - 0.000071F^2$		0.949	0.008
	(0.34) (0.0017) (0.0080) (0.033) (0.000037)			
	$+ 0.000068W^2 - 0.00013P^2$			
	(0.000087) (0.033)			
	Heat capacity, Cp (kJ/kg °C)	26		
Cp1	$C_p = 4.408 + 0.02742W - 0.01973\ F - 0.08368\mathrm{P}$		0.506	0.124
	(1.052) (0.01647) (0.01009) (0.04409)			
Cp2	$C_p = 14.040 - 0.128W - 0.187F - 0.388P + 0.00097W*F$		0.612	0.118
	(6.45) (0.143) (0.098) (0.22) (0.0027)			
	$+ 0.00476W*P + 0.00425F*P$			
	(0.0039) (0.0060)			
	Thermal diffusivity, $\alpha \times 10^{-7}$ m^2/s)	31		
α1	$\alpha = 1.35 - 0.0143W - 0.0103F + 0.0246P$		0.335	0.082
	(0.61) (0.0095) (0.0061) (0.0232)			
α2	$\alpha = -10.3 + 0.155W + 0.177F + 0.453P + 0.00061W*F$		0.559	0.071
	(3.67) (0.0751) (0.0546) (0.1306) (0.00106)			
	$- 0.00634W*P - 0.0076F*P$			
	(0.00225) (0.00283)			

Note: F is percent fat content with a range from 8–37%, W is percent moisture content with a range from 30–60%, and P is protein content with a range from 22–36%. The values in parenthesis are standard errors for each coefficient. n is number of observations, R^2 is a measure of the extent of a linear relationship between two data sets, and SE is equation's standard error.

all other independent variables remain unchanged. However, this conventional interpretation can not be applied here because fat, moisture, and protein contents of cheese add up to almost 100%, and that all three are used as independent variables in the regression model. One cannot increase one independent variable and at the same time maintain the other variables unchanged. A better way to verify the robustness of the model is to ask, for example: How does the thermal conductivity of Cheddar cheese change if one increases the fat content by one percentage point and simultaneously reduces the moisture content by one percentage point? Table VI shows the result of the pairwise substitution test conducted to answer at the question. It was concluded that Model k1 was robust because all three pairwise substitution tests were significant and the direction of changes was compatible with theoretical prediction and with previous results. The result for moisture-for-protein substitution was not as significant as others and the direction of change were not as expected.

Two models were considered for heat capacity prediction of Cheddar cheese (Model Cp1 and Cp2 in Table V). Compared with the basic model (Model Cp1), the extended model (Model Cp2) has lower *t*-statistics and a modest increase in R^2. With the same reasoning as before, the basic model is preferred over the extended model. The interpretation of the regression proceeds analogously to the interpretation of the thermal conductivity model discussed above. The results of pairwise substitution tests were significant unless for fat-for-protein substitution. The latest result was expected since the heat capacity values of protein and fat are almost the same, thus substitution of one component to the other will not affect the overall value.

The development of empirical models predicting thermal diffusivity suffers from the poor quality of measured data. Measurements of thermal diffusivity were repeated three times for each observation. The coefficient of variation of the three replications ranged from 15–25%, suggesting measurement errors are high. Two models were considered for thermal diffusivity

Table VI. Robustness Test of the Empirical Models for Thermal Properties Prediction

X for *Y*[a]	Coefficient[b]	Standard Error[c]	*t*-Statistics[d]
Thermal conductivity			
Fat for moisture	−0.00299	0.00102	2.93
Fat for protein	−0.0081	0.0022	3.68
Moisture for protein	−0.0051	0.002303	2.21
Heat capacity			
Fat for moisture	0.04715	0.0165	2.85
Fat for protein	0.064	0.0454	1.41
Moisture for protein	0.111	0.04703	2.36

Note:
[a] *X* for *Y* means increase of component *X* by one percentage point and simultaneously reduce the component *Y* one percentage point.
[b] Calculated by substracting the coefficient of component *Y* from the coefficient of component *X* (from Table 6). This number indicates the change of the dependent variable (thermal property) if one does pairwise substitution of *X* for *Y*.
[c] Calculated as the square root of the sum of the squares of the standard errors of the coefficient estimates of component *X* and *Y*.
[d] Calculated by dividing the coefficient to the standard error. Value greater than 2.05 indicates that the result is statistically significant at 5% level.

prediction of Cheddar cheese (Model $\alpha1$ and $\alpha2$ in Table V). Compared to the simple model (Model $\alpha1$), both *t*-ratios and R^2 of Model $\alpha2$ improve substantially. Even though the Model $\alpha2$ contains a number of explanatory variables which is large given the number of available observations, its performance improvements are large enough to justify its selection.

Model $\alpha2$ carries interaction terms and that makes the previous way of interpretation can not be used. The robustness of this model will be determined by comparing the thermal diffusivity value at the mean of all explanatory variables (20.75% fat, 44.98% moisture, and 29.26% protein), which is $1.202 \times 10^{-7}\,m^2/s$, with the change of the thermal diffusivity value when increasing one component by one percentage point and at the same time decreasing other component by the same percentage point. Fat-for-moisture substitution resulted in a decrease of thermal diffusivity by $2.68 \times 10^{-5} \times 10^{-7}\,m^2/s$, which is negligible if compared to the value of $1.202 \times 10^{-7}\,m^2/s$. Thus, the model indicates a substitution of water with fat does not change thermal diffusivity. Substitution of protein-for-fat resulted in a decrease in thermal diffusivity by $0.0198 \times 10^{-7}\,m^2/s$; a small number but can not be ignored. Water-for-protein substitution resulted in a decrease of thermal diffusivity by $0.0216 \times 10^{-7}\,m^2/s$ or 1.8%. This result indicates a substantial impact of water-for-protein substitution on thermal diffusivity.

Comparison of Empirical Model with Literature Models

Table VII shows the result of goodness-of-fit tests for thermal conductivity prediction using theoretical models in Table I. Almost all of them underpredicted the thermal conductivity of Cheddar cheese, except for one model (Kopelman-parallel model) which also happened to be the best theoretical model for predicting thermal conductivity of Cheddar cheese.

The predictability of the Maxwell-Eucken (Eq. (3)) [44, 45, 14, 15, 7] model was good noting that it is theoretically only valid for dilute suspensions. This model was still chosen because of work by Sakiyama [46] and Morley and Miles [47] indicated that the equation still yielded good result over quite a wide concentration range. The deviations of first Kopelman model (Eq. (4)) [16, 48] were caused by the assumption of negligible thermal conductivity value of the dispersed phase. This model works well only when the thermal conductivity of the continuous phase is much larger than that of the dispersed phase. The second Kopelman model (Eq. (5)) includes the thermal conductivity of the dispersed phase resulted a better prediction model.

The next set of models tested assumes two-component systems with layered components constituting the discontinuous phase and the thermal conductivity was measured: (a) parallel to the layers (Eq. (6)) and (b) perpendicular to the component layers (Eq. (7)). The first assumption (parallel)

Table VII. Results of Goodness-of-Fit Test for Thermal Conductivity Prediction Using Models from Table I

Model	Model No.	|% Error|[a]	Mean (|% Error|)	Predictibility[b]
Maxwell-Eucken		0.5–13.8	5.3	−5.2
Kopelman	1	3–25.8	14	−14
Kopelman	2	0.1–11.3	3.4	−2.7
Kopelman	3	0.1–10.1	2.5	0.6
Kopelman	4	21–43	32	−32
Poppendick	1	0.15–23.6	10	−5.6
Poppendick	2	18.6–52	42	−42
Series		29–50	42	−42
Parallel		0.1–16.4	7.5	−6

Note:
[a] Absolute percent errors are defined as the absolute difference between the empirically measured value and the value predicted by the theoretical model, divided by the empirically measured value.
[b] Measured as the average of the difference between empirically measured values and values predicted by the empirical model, divided by the empirically measured values. Minus value indicates that the model unpredicts the empirical data.

yielded the best prediction for thermal conductivity of Cheddar cheese. Slightly different models from the Kopelman models were proposed by Poppendick et al. [17]. He treated food treated as if it is laminated into layers consisting of its components which are considered to be electrical conductors, which are arranged in a parallel system (Eq. (8)) and in perpendicular system (Eq. (9)). Similar to the pattern shown in the Kopelman models, the perpendicular model fared worse than its counterpart. The last two models tested were the series model and the parallel model [13]. Again, the parallel model fared better than its counterpart, the series model.

The second part of this comparison was to compare the predictive ability of empirically built models of this study with other empirical models from the literature. The Choi & Okos and the Sweat model underpredicted the thermal conductivity values of Cheddar cheese by 13% and 10%; whereas Model k1 achieved an average percent error of 2.4%. Thus, the thermal conductivity model of Cheddar cheese developed in this study outperforms alternatives available in the literature. The Choi & Okos and the Heldman & Singh model could predict the heat capacity of Cheddar cheese reasonably well. The Choi & Okos model overpredicted the out-of-sample data by 5.1%, the Heldman & Singh model underpredicted by 3.5%, and Model Cp1 overpredicted by 5.5%. Thus, given the results of this comparison it is reasonable to conclude the Heldman & Singh model has the best predictive ability of heat capacity of Cheddar cheese. The low data quality was an obstacle in developing a useful model predicting thermal diffusivity.

Nonetheless, a comparison between the predictive ability of the thermal diffusivity model and other empirical models available in the literature is presented here. Both the Choi & Okos and the Hermans model were able to predict thermal diffusivity of Cheddar cheese with reasonable accuracy. Their models underpredicted the thermal diffusivity values of Cheddar cheese by only 4.8% and 2.5%. Model $\alpha 2$ yielded an average percent error of 2%. In summary, the predictive ability of the three thermal diffusivity models tested – two literature models and the one developed in this study – is approximately equal and no model distinguished itself as clearly superior.

CONCLUSIONS

From the results of this study, it may be concluded that:

1. Thermal conductivity and heat capacity of Cheddar cheeses ranged from 0.354–0.481 W/m °C and from 2.444–3.096 kJ/kg °C, respectively in the composition range of 30–60% moisture, 8–37% fat, and 22–36% protein,
2. Both thermal conductivity and heat capacity increased with moisture and protein content and decreased with fat content,
3. Thermal diffusivity ranged from 1.07×10^{-7}–$1.53 \times 10^{-7}\,m^2/s$ and there was no significant relationship between thermal diffusivity and moisture, fat and protein content of cheese,
4. The differences in thermal properties (at the 10% level) with age (0 to 28 week) were not statistically significant,
5. Models predicting thermal properties of cheese as a function composition were developed, and
6. The developed thermal conductivity models better predicted the thermal conductivity of Cheddar cheese than the published literature models.

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